Entire and meromorphic functions
Problem set 11 (due Tuesday, February 5)

1. Let f be meromorphic and let $b \in \mathbb{C}$ such that f has infinitely many b-points $a_1, a_2, \ldots$. Show that, for all $q \in \mathbb{N}$,

$$m\left(r, \frac{1}{f \circ g - b}\right) \geq \sum_{j=1}^{q} m\left(r, \frac{1}{g - a_j}\right) - \mathcal{O}(1)$$

as $r \to \infty$. Conclude that

$$\lim_{r\to\infty} \frac{T(r, f \circ g)}{T(r, g)} = \infty.$$

2. Let a_0, a_1, a_2 be rational functions and let f be a transcendental meromorphic solution of the differential equation

$$f' = a_0 + a_1 f + a_2 f^2.$$

   (a) Suppose that $a_2 \neq 0$. Show that $m(r, f) = o(T(r, f))$ as $r \to \infty$, $r \notin E$, for some subset E of $[0, \infty)$ of finite measure. Conclude that $\delta(\infty, f) = 0$.

   (b) Show that f has only finitely many multiple poles and deduce that $\Theta(\infty, f) = 0$.

   (c) Let $c \in \mathbb{C}$ such that $a_0 + ca_1 + c^2 a_2 \neq 0$. Show that $\Theta(c, f) = 0$.

Hint. (a) Write the equation as

$$f = \frac{1}{a_2}\left(\frac{f'}{f} - \frac{a_0}{f} - a_1\right)$$

in order to estimate $\log^+ |f(re^{i\theta})|$.

(c) Find a Riccati equation solved by $g = 1/(f - c)$.