Entire and meromorphic functions
Problem set 6 (due Friday, December 8)

1. Prove the Cauchy criterion for infinite products: For a sequence (a_k) in $\mathbb{C}$ the infinite product $\prod_{j=1}^{\infty} a_j$ converges if and only if for every $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that
$$\left| \prod_{j=m}^{n} a_j - 1 \right| < \varepsilon \quad \text{for } n \geq m \geq N.$$

2. Let D be a domain and let (f_k) be a sequence of functions which are holomorphic in D. Suppose that the infinite product
$$f(z) = \prod_{k=1}^{\infty} f_k(z)$$
converges locally uniformly in D. Show that
$$\frac{f'(z)}{f(z)} = \sum_{k=1}^{\infty} \frac{f_k'(z)}{f_k(z)}$$
for $z \in \mathbb{C} \backslash f^{-1}(0)$ and that the convergence is locally uniform (in D).

3. Let $\tau \in \mathbb{C}$ with $\operatorname{Im} \tau > 0$ and defined $\Lambda = \{m + n\tau \colon m, n \in \mathbb{Z}\}$. Show that the exponent of convergence as well as the genus of Λ is 2.