Prof. Dr. W. Bergweiler Winter semester 2017/18

Entire and meromorphic functions
Problem set 3 (due Friday, November 17)

1. Let f be meromorphic and $\rho > 0$. Show that $N(r, f) \sim r^\rho$ as $r \to \infty$ if and only if $n(r, f) \sim \rho r^\rho$ as $r \to \infty$.

2. Let $P(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$ be a polynomial, with $n \geq 1$. Show that

$$T(r, \exp \circ P) = \frac{r^n}{\pi} + \mathcal{O}(r^{n-1})$$

as $r \to \infty$.

3. Show that $m(r, \tan) = \mathcal{O}(1)$. Actually, $m(r, \tan) = o(1)$ as $r \to \infty$, and if you are more ambitious you can also show this.

Hint: Show first that there exists a constant $C > 0$ such that

$$|\tan z| \leq C \max\left\{1, \frac{1}{|\operatorname{Im} z|}\right\}$$

for $z \in \mathbb{C}\backslash\mathbb{R}$. Actually,

$$|\tan z| \leq 1 + \frac{1}{|\operatorname{Im} z|}$$

for $z \in \mathbb{C}\backslash\mathbb{R}$.