Complex dynamics
Problem set 7 (due Tuesday, June 7)

1. For $\lambda > 0$ let $f_\lambda(z) = \lambda(z^2 - 1)$; cf. Example 2.2 of the lecture. Show that

$$\dim J(f_\lambda) = (1 + o(1))\frac{\log 2}{\log \lambda}$$

as $\lambda \to \infty$.

2. A subset X of $\mathbb{R}^n$ is called *porous* if there exists $c > 0$ such that for every $x \in \mathbb{R}^n$ and every $r > 0$ there exists $y \in \mathbb{R}^n$ such that $B(y, cr) \subset B(x, r) \backslash X$.

   Show that if X is a porous subset of $\mathbb{R}^n$, then $\dim X < n$.

3. Let f be a polynomial od degree $d \geq 2$ and let $M = \sup_{z \in J(f)} |f'(z)|$. Show that

$$\dim J(f) \geq \frac{\log d}{\log M}.$$

   Hint. The result is easier to prove if $J(f)$ contains no critical points, so you may consider this case first (or decide to consider only this case). Then the result can be deduced from a slight generalization of Theorem 2.3. (This generalization can be obtained with the same proof, so you do not have to repeat the proof, but should say what the result that you need is.)

   If $J(f)$ contains a critical point, use that $J(f) = J(f^n)$ for all $n \in \mathbb{N}$.