Prof. Dr. W. Bergweiler Winter semester 2015/16

Complex dynamics
Problem set 3 (due Monday, November 16)

Definition: A Möbius transformation of the form $z \mapsto z + c$ with $c \in \mathbb{C}$ is called a *translation*, one of the form $z \mapsto cz$ with $c \in \mathbb{C}$, $|c| = 1$, is called a *rotation* and one of the form $z \mapsto cz$ with $c \in \mathbb{R}$, $c > 0$, is called a *dilation*. The Möbius transformation given by $z \mapsto 1/z$ is called the *inversion*.

1. Show that every Möbius transformation can be written as the composition of translations, rotations, dilations and inversions.

2. Let C be the set of all circles in $\mathbb{C}$ and let L be the set of all straight lines in $\mathbb{C}$, with the point ∞ added to each such straight line. Show that if T is a Möbius transformation, then $T(C \cup L) = C \cup L$.

   Hint: Use Problem 1.

3. Let $\mathcal{F}$ be a normal family of functions holomorphic in a domain G containing 0. Suppose that $f(0) = 0$ and $f'(0) = 1$ for all $f \in \mathcal{F}$. Show that there exists $r > 0$ such that $D(0, r) \subset f(G)$ for all $f \in \mathcal{F}$.

4. Let U, V, W be domains in $\widehat{\mathbb{C}}$ and let $g\colon U \to V$ and $h\colon W \to \widehat{\mathbb{C}}$ be meromorphic. Let $\mathcal{F}$ be a family of meromorphic functions $f\colon V \to W$. Suppose that $\mathcal{F}$ is normal.

   (a) Is $\{f \circ g\colon f \in \mathcal{F}\}$ normal?

   (b) Is $\{h \circ f\colon f \in \mathcal{F}\}$ normal?