Complex dynamics
Problem set 2 (due Monday, November 9)

1. Let $\varphi\colon S^2 \to \widehat{\mathbb{C}}$ be the stereographic projection. Verify the formulae for φ and φ^{-1} given in the lecture.

2. Verify the formula for the chordal distance given in the lecture.

3. Let $a_1, a_2, a_3, b_1, b_2, b_3 \in \widehat{\mathbb{C}}$ with $a_j \neq a_k$ and $b_j \neq b_k$ for $j \neq k$. Show that there exists a Möbius transformation satisfying $T(a_j) = b_j$ for all $j \in \{1, 2, 3\}$.

   Hint: Consider the special case $b_1 = 0$, $b_2 = 1$ and $b_3 = \infty$ first.

4. Show that every Möbius transformation is conjugate to a Möbius transformation of the form $z \mapsto kz$ where $k \in \mathbb{C}\backslash\{0\}$ or $z \mapsto z + 1$; that is, show that for every Möbius transformation f there exists a Möbius transformation T such that $g := T^{-1} \circ f \circ T$ has the form $g(z) = kz$ or $g(z) = z + 1$.

   Hint: Show first that a Möbius transformation different from the identity has one or two fixed points.