Complex dynamics
Problem set 10 (due Monday, January 25)

1. Let $f\colon \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$,
$$f(z) = \frac{(z-2)^2}{z^2}.$$
Determine $\operatorname{sing}(f^{-1})$ and the orbits of the points in $\operatorname{sing}(f^{-1})$. Use this to show that f has no attracting periodic points.

2. For an entire or rational function f denote by $C(f)$ the set of critical values of f. Show that if f and g are both rational or both entire, then
$$C(f \circ g) \subset f(C(g)) \cup C(f),$$
with equality if f and g are rational, while the inclusion may be strict if f and g are entire.

   Denote by $A(f)$ the set of asymptotic values of an entire function. Show that if f and g are entire, then
$$A(f \circ g) \subset f(A(g)) \cup A(f).$$

3. Let P be a non-constant polynomal and let
$$N_P(z) = z - \frac{P(z)}{P'(z)}$$
be the associated Newton function. Suppose that N_P has an attracting periodic point z_0 of period $m \geq 2$. Show that
$$\bigcup_{j=0}^{m-1} A^*(N_P^j(z_0))$$
contains a zero of P''.

   Use this to show that for $P(z) = z^3 - 2z + 2$ the function N_P has a superattractive periodic point of period 2.

4. Let $P(z) = (z-1)(z+1)(z-a)$. Then $P''(a/3) = 0$. Bei Problem 3, N_P has no attracting periodic points besides the zeros of P, if $a/3$ is in the attracting basin of one of the zeros of P.

   Make computer graphics showing the corresponding a-values. Consider in particular the range given by $|\operatorname{Re} a| \leq 0.2$ und $|\operatorname{Im} a - 4.55| \leq 0.2$