Analytic number theory
Problem set 9 (due Friday, June 29)

1. Show that
$$\frac{\zeta'(0)}{\zeta(0)} = \log(2\pi).$$

   Hint. Use the functional equation of ζ and Problem 1 of Problem set 5.

2. Let $m \in \mathbb{N}$. Show that
$$\zeta'(-2m) = (-1)^m \frac{\zeta(2m+1)(2m)!}{2^{2m+1}\pi^{2m}}.$$

   Hint. Combine the definition of the derivative with the functional equation of ζ and Theorem 8.7 (which says that $\zeta(-2m) = 0$).

3. Show that
$$\pi^{-z/2}\Gamma\left(\tfrac{1}{2}z\right)\zeta(z) = \pi^{-(1-z)/2}\Gamma\left(\tfrac{1}{2}(1-z)\right)\zeta(1-z).$$