Prof. Dr. W. Bergweiler Winter semester 2017/18

Analytic number theory
Problem set 1 (due Friday, April 20)

1. The recursion formula for the Fibonacci numbers can also be written in the form
$$\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = A \begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix}$$
with
$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$$
Use linear algebra to compute A^n and deduce the formula for F_n obtained in the lecture from this.

2. Let $p \in \mathbb{C}$ and let $(a_n)_{n\geq 0}$ be defined by $a_0 = 0$, $a_1 = 1$ and $a_n = 2pa_{n-1} - a_{n-2}$ for $n \geq 2$.

   Determine the generating function of (a_n) and use this to find a closed-form expression for a_n.

3. Let $(a_n)_{n\geq 0}$ be a sequence of complex numbers and let
$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$
be its generating function. Let $(s_n)_{n\geq 0}$ and $(t_n)_{n\geq 0}$ be defined by
$$s_n = \sum_{n=0}^{n} a_k$$
and $t_n = na_n$.

   Determine the generating functions of $(s_n)_{n\geq 0}$ and $(t_n)_{n\geq 0}$.