

```
> f := (x, y) -> x*y - x^3 - y^2;
```

$$f := (x, y) \rightarrow xy - x^3 - y^2$$

```
> f(x, y);
```

$$xy - x^3 - y^2$$

```
> f1 := diff(f(x, y), x);
```

$$f1 := y - 3x^2$$

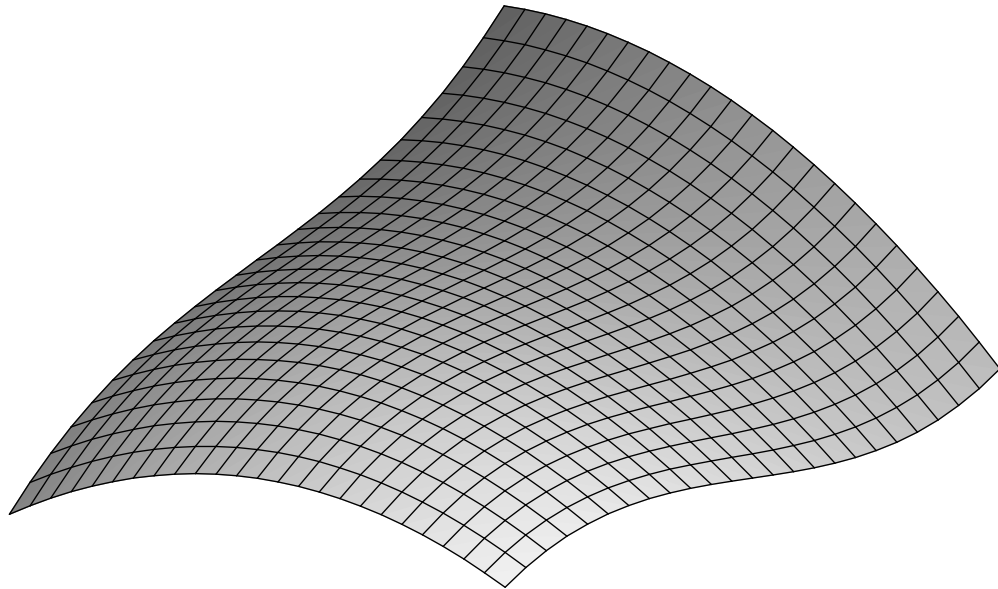
```
> f2 := diff(f(x, y), y);
```

$$f2 := x - 2y$$

```
> s := solve({f1=0, f2=0}, {x, y});
```

$$s := \{y=0, x=0\}, \{y=\frac{1}{12}, x=\frac{1}{6}\}$$

```
> plot3d(f(x, y), x=-1..1, y=-1..1);
```



```
> fxx:=diff(f(x,y),x,x);  
                fxx:=-6 x  
> fxy:=diff(f(x,y),x,y);  
                fxy:= 1  
> fyy:=diff(f(x,y),y,y);  
                fyy:=-2  
> diff(f(x,y),y,x);  
                1  
> d:=fxx*fyy-fxy^2;  
                d:= 12 x- 1  
> subs(s[1],d);  
                -1
```

```
> subs(s[2],d);
```

1

```
> subs(s[1],f(x,y));
```

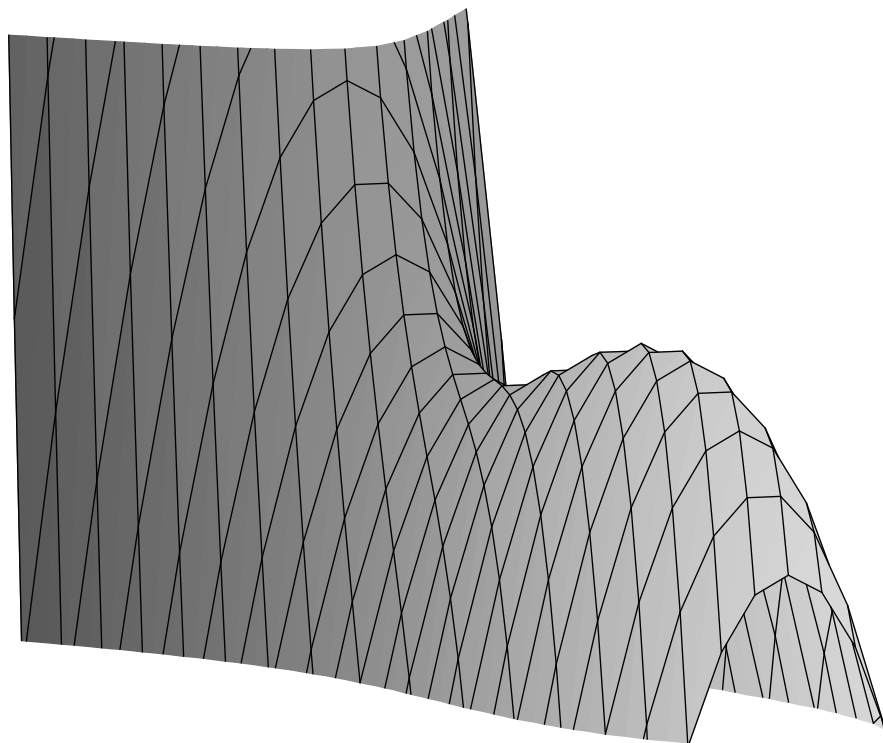
0

```
> subs(s[2],f(x,y));
```

$\frac{1}{432}$

432

```
> plot3d(f(x,y),x=-0.3..0.3,y=-0.3..0.3,view=-0.01..0.01);
```



```
> with(linalg);
```

Warning, the protected names norm and trace have been redefined and unprotected

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

> f(x,y);

$$xy - x^3 - y^2$$

> grad(f(x,y), [x,y]);

$$[y - 3x^2, x - 2y]$$

> solve(%, {x,y});

Error, (in solve) invalid arguments

```
> convert(grad(f(x,y),[x,y]),set);
```

$$\{y-3x^2, x-2y\}$$

```
> s:=solve(%,{x,y});
```

$$s := \{y=0, x=0\}, \{y=\frac{1}{12}, x=\frac{1}{6}\}$$

```
> he:=hessian(f(x,y),[x,y]);
```

$$he := \begin{bmatrix} -6x & 1 \\ 1 & -2 \end{bmatrix}$$

```
> eigenvals(%);
```

$$-3x-1+\sqrt{9x^2-6x+2}, -3x-1-\sqrt{9x^2-6x+2}$$

```
> he1:=subs(s[1],matrix(he));
```

$$he1 := \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

```
> eigenvals(%);
```

$$-1+\sqrt{2}, -1-\sqrt{2}$$

```
> definite(he1,'positive_def');
```

false

```
> definite(he1,'negative_def');
```

false

```
> f:=(x,y,z)->(x^2-2*x*y+z^2)*exp(x-y-z^2);
```

$$f := (x, y, z) \rightarrow (x^2 - 2xy + z^2) e^{(x-y-z^2)}$$

```
> convert(grad(f(x,y,z),[x,y,z]),set);
```

$$\{2z e^{(x-y-z^2)} - 2(x^2 - 2xy + z^2)z e^{(x-y-z^2)},$$

$$-2 x e^{(x-y-z^2)} - (x^2 - 2 x y + z^2) e^{(x-y-z^2)},$$

$$(2 x - 2 y) e^{(x-y-z^2)} + (x^2 - 2 x y + z^2) e^{(x-y-z^2)} \}$$

> s := solve(% , {x, y, z}) ;

s := {z = 0, y = 0, x = 0}, {z = 0, x = -2, y = 0},

$$\left\{ z = \frac{1}{2} \text{RootOf}(-3 + _Z^2, \text{label} = _L1), x = \frac{-1}{2}, y = 0 \right\}$$

> t := allvalues(s[3]) ;

$$t := \left\{ z = \frac{1}{2} \sqrt{3}, x = \frac{-1}{2}, y = 0 \right\}, \left\{ z = -\frac{1}{2} \sqrt{3}, x = \frac{-1}{2}, y = 0 \right\}$$

> he := hessian(f(x, y, z), [x, y, z]) ;

he :=

$$\left[2 e^{(x-y-z^2)} + 2 (2 x - 2 y) e^{(x-y-z^2)} + (x^2 - 2 x y + z^2) e^{(x-y-z^2)}, \right.$$

$$\left. -2 e^{(x-y-z^2)} - 2 x e^{(x-y-z^2)} - (2 x - 2 y) e^{(x-y-z^2)} \right.$$

$$\left. - (x^2 - 2 x y + z^2) e^{(x-y-z^2)}, 2 z e^{(x-y-z^2)} - 2 (2 x - 2 y) z e^{(x-y-z^2)} \right.$$

$$\left. - 2 (x^2 - 2 x y + z^2) z e^{(x-y-z^2)} \right]$$

$$\left[-2 e^{(x-y-z^2)} - 2 x e^{(x-y-z^2)} - (2 x - 2 y) e^{(x-y-z^2)} \right.$$

$$\left. - (x^2 - 2 x y + z^2) e^{(x-y-z^2)}, \right.$$

$$\left. 4 x e^{(x-y-z^2)} + (x^2 - 2 x y + z^2) e^{(x-y-z^2)}, \right.$$

$$\left. -2 z e^{(x-y-z^2)} + 4 x z e^{(x-y-z^2)} + 2 (x^2 - 2 x y + z^2) z e^{(x-y-z^2)} \right]$$

$$\begin{aligned}
& [2z e^{(x-y-z^2)} - 2(2x-2y)z e^{(x-y-z^2)} \\
& - 2(x^2 - 2xy + z^2)z e^{(x-y-z^2)}, \\
& -2z e^{(x-y-z^2)} + 4xz e^{(x-y-z^2)} + 2(x^2 - 2xy + z^2)z e^{(x-y-z^2)}, \\
& 2e^{(x-y-z^2)} - 8z^2 e^{(x-y-z^2)} - 2(x^2 - 2xy + z^2)e^{(x-y-z^2)} \\
& + 4(x^2 - 2xy + z^2)z^2 e^{(x-y-z^2)}]
\end{aligned}$$

> he1:=simplify(subs(s[1],matrix(he)));

$$he1 := \begin{bmatrix} 2 & -2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

> eigenvals(he1);

$$2, 1 + \sqrt{5}, 1 - \sqrt{5}$$

> he2:=simplify(subs(s[2],matrix(he)));

$$he2 := \begin{bmatrix} -2e^{(-2)} & 2e^{(-2)} & 0 \\ 2e^{(-2)} & -4e^{(-2)} & 0 \\ 0 & 0 & -6e^{(-2)} \end{bmatrix}$$

> eigenvals(he2);

$$-6e^{(-2)}, -3e^{(-2)} + \sqrt{9(e^{(-2)})^2 - 4e^{(-4)}}, -3e^{(-2)} - \sqrt{9(e^{(-2)})^2 - 4e^{(-4)}}$$

> evalf(%);

$$-.8120116992, -.1033869567, -.7086247425$$

> definite(he2, 'negative_def');

$$-2 e^{(-2)} < 0 \text{ and } -4 (e^{(-2)})^2 < 0 \text{ and } -24 (e^{(-2)})^3 < 0$$

> he3:=simplify(subs(t[1],matrix(he)));

$$he3 := \begin{bmatrix} e^{(-5/4)} & -e^{(-5/4)} & \sqrt{3} e^{(-5/4)} \\ -e^{(-5/4)} & -e^{(-5/4)} & -\sqrt{3} e^{(-5/4)} \\ \sqrt{3} e^{(-5/4)} & -\sqrt{3} e^{(-5/4)} & -3 e^{(-5/4)} \end{bmatrix}$$

> eigenvals(he3);

$$\frac{1}{3} (-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3))^{(1/3)} - 3 \left(-\frac{8}{3} e^{(-5/2)} - (e^{(-5/4)})^2 \right) / (-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3))^{(1/3)} - e^{(-5/4)}, -\frac{1}{6} (-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3))^{(1/3)} - 3$$

$$\begin{aligned}
& \left. \begin{aligned}
& + \frac{3}{2} \left(-\frac{8}{3} e^{(-5/2)} - (e^{(-5/4)})^2 \right) / \left(-108 e^{(-5/2)} e^{(-5/4)} \right. \\
& + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 \\
& + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} \\
& - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right)^{(1/3)} - e^{(-5/4)} + \frac{1}{2} I \sqrt{3} \left(\frac{1}{3} (\right. \\
& -108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(\\
& 384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} \\
& - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right)^{(1/3)} + 3 \\
& \left(-\frac{8}{3} e^{(-5/2)} - (e^{(-5/4)})^2 \right) / \left(-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} \right. \\
& - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 \\
& + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right)^{(1/3)} \\
& \left. \right)^{(1/3)} \left. \right) - \frac{1}{6} \left(-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(\right. \\
& 384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} \\
& - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right)^{(1/3)} + \frac{3}{2}
\end{aligned}
\right.
\end{aligned}$$

$$\begin{aligned}
& \left(-\frac{8}{3} e^{(-5/2)} - (e^{(-5/4)})^2 \right) / \left(-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} \right. \\
& - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 \\
& + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right) \\
& (1/3) \\
& - e^{(-5/4)} - \frac{1}{2} I \sqrt{3} \left(\frac{1}{3} (-108 e^{(-5/2)} e^{(-5/4)} + 162 e^{(-15/4)} \right. \\
& - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 \\
& + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right) \\
& (1/3) \\
& + 3 \left(-\frac{8}{3} e^{(-5/2)} - (e^{(-5/4)})^2 \right) / \left(-108 e^{(-5/2)} e^{(-5/4)} \right. \\
& + 162 e^{(-15/4)} - 27 (e^{(-5/4)})^3 + 6 I \text{sqrt}(384 (e^{(-5/2)})^3 \\
& + 108 (e^{(-5/2)})^2 (e^{(-5/4)})^2 + 972 e^{(-5/2)} e^{(-5/4)} e^{(-15/4)} \\
& - 729 (e^{(-15/4)})^2 + 243 e^{(-15/4)} (e^{(-5/4)})^3 \left. \right) (1/3)
\end{aligned}$$

> evalf(%);

.6887709965 - .1 10⁻⁹ I, -1.209530831 - .1732050808 10⁻⁹ I,
-.3387545561 + .1732050808 10⁻⁹ I

> maximize(f(x,y,z), x=-3..3, y=-3..3, z=-3..3, location=true);

27 e⁶, {[{z=0, y=-3, x=3}, 27 e⁶]}

```

[ >
[ > A:=matrix(2,3,[4,5,6,3,4,2]);
      A:= $\begin{bmatrix} 4 & 5 & 6 \\ 3 & 4 & 2 \end{bmatrix}$ 
[ > b:=vector([5,6]);
      b := [5,6]
[ > linsolve(A,b);
       $[-10 - 14 \_t_1, 9 + 10 \_t_1, \_t_1]$ 
[ > B:=matrix(2,2,[3,7,u,v]);
      B:= $\begin{bmatrix} 3 & 7 \\ u & v \end{bmatrix}$ 
[ > C:=multiply(A,B);
Error, (in multiply) non matching dimensions for
vector/matrix product
[ > C:=multiply(B,A);
      C:= $\begin{bmatrix} 33 & 43 & 32 \\ 4u+3v & 5u+4v & 6u+2v \end{bmatrix}$ 
[ > multiply(B,B,A);
       $\begin{bmatrix} 99+28u+21v & 129+35u+28v & 96+42u+14v \\ 33u+4vu+3v^2 & 43u+5vu+4v^2 & 32u+6vu+2v^2 \end{bmatrix}$ 
[ > transpose(A);
       $\begin{bmatrix} 4 & 3 \\ 5 & 4 \\ 6 & 2 \end{bmatrix}$ 
[ > matrix(A);

```

$$\begin{bmatrix} 4 & 5 & 6 \\ 3 & 4 & 2 \end{bmatrix}$$

> matrix(B);

$$\begin{bmatrix} 3 & 7 \\ u & v \end{bmatrix}$$

> charpoly(B,x);

$$x^2 - xv - 3x + 3v - 7u$$

> factor(%);

$$x^2 - xv - 3x + 3v - 7u$$

> eigenvals(B);

$$\frac{3}{2} + \frac{1}{2}v + \frac{1}{2}\sqrt{9 - 6v + v^2 + 28u}, \frac{3}{2} + \frac{1}{2}v - \frac{1}{2}\sqrt{9 - 6v + v^2 + 28u}$$

> eigenvectors(B);

$$\left[\frac{3}{2} + \frac{1}{2}v + \frac{1}{2}\sqrt{9 - 6v + v^2 + 28u}, 1, \right.$$

$$\left. \left\{ \left[1, -\frac{3}{14} + \frac{1}{14}v + \frac{1}{14}\sqrt{9 - 6v + v^2 + 28u} \right] \right\} \right], \left[\right.$$

$$\frac{3}{2} + \frac{1}{2}v - \frac{1}{2}\sqrt{9 - 6v + v^2 + 28u}, 1,$$

$$\left. \left\{ \left[1, -\frac{3}{14} + \frac{1}{14}v - \frac{1}{14}\sqrt{9 - 6v + v^2 + 28u} \right] \right\} \right]$$

> minpoly(B,x);

$$3v - 7u + (-3 - v)x + x^2$$

> A:=matrix(3,3,[0,0,0,0,0,0,0,0,1]);

$$A := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> eigenvals(A);
```

0, 0, 1

```
> charpoly(A, x);
```

$x^2(x-1)$

```
> eigenvectors(A);
```

[1, 1, {[0, 0, 1]}], [0, 2, {[0, 1, 0], [1, 0, 0]}]

```
> F := (i, j) -> 1 / (i + j);
```

$$F := (i, j) \rightarrow \frac{1}{i+j}$$

```
> F(i, j);
```

$$\frac{1}{i+j}$$

```
> F(3, 4);
```

$$\frac{1}{7}$$

```
> M := matrix(6, 6, F);
```

$$M := \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \end{bmatrix}$$

> det(M);

1

172153600393420800000

>