

```

[ > restart;

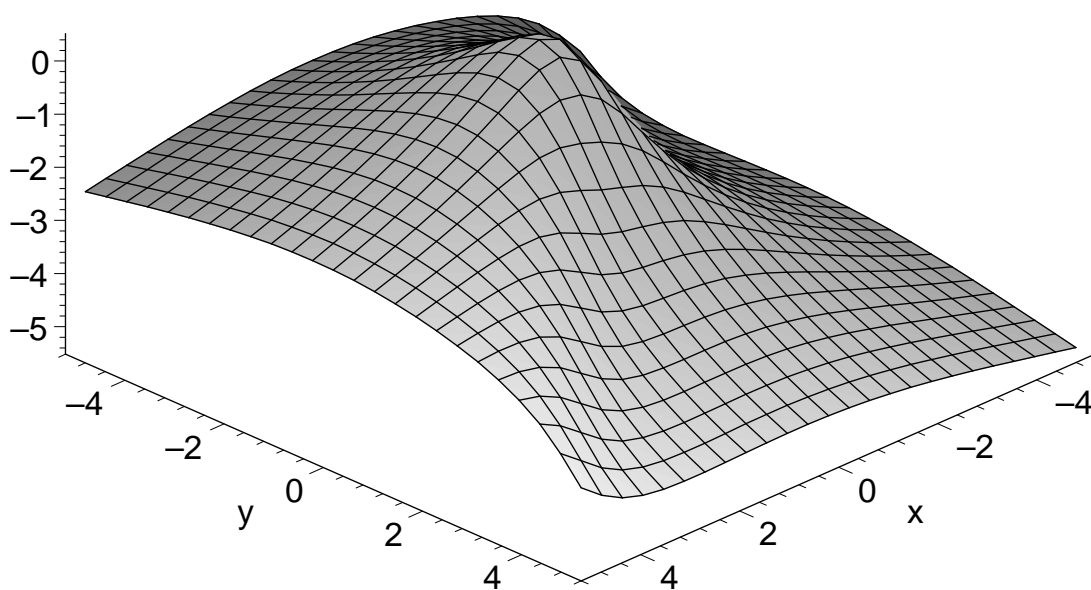
[ > p:=x^7+x^6+x^5+x^4-4*x^3-4*x^2-4*x-4;
      p := x^7 + x^6 + x^5 + x^4 - 4 x^3 - 4 x^2 - 4 x - 4
[ > factor(p);
      (x + 1)(x^2 - 2)(x^2 + 2)(x^2 + 1)
[ > factor(p, 2^(1/2));
      (x^2 + 1)(x^2 + 2)(x + sqrt(2))(x - sqrt(2))(x + 1)
[ > factor(p, I);
      -(x^2 - 2)(x^2 + 2)(-x + I)(x + I)(x + 1)
[ > factor(p, {I, 2^(1/2)});
      (x - I)(x + I)(x + sqrt(2))(-x + sqrt(2))(x + I sqrt(2))(-x + I sqrt(2))(x + 1)
[ > factor(p, (-2)^(1/2));
      (x^2 + 1)(x^2 - 2)(x + sqrt(-2))(x - sqrt(-2))(x + 1)
[ > factor(p, real);
(x + 1.414213562)(x + 1.)(x - 1.414213562)(x^2 + 1.999999999)
(x^2 + 1.000000000)
[ > factor(p, complex);
(x + 1.414213562)(x + 1.)(x + 1.414213562 I)
(x + 1.000000000 I)(x - 1. I)(x - 1.414213562 I)
(x - 1.414213562)
[ > Factor(p) mod 2;
      (x + 1)^3 x^4
[ > Factor(p) mod 3;
      (x + 1)^2 (x^2 + 1)^2 (x + 2)
[ >
-

```

```

>
> f := (x, y) -> arctan(x - y) - ln(1 + x^2 + y^2);
      f := (x, y) -> arctan(x - y) - ln(1 + x^2 + y^2)
> plot3d(f(x, y), x = -5..5, y = -5..5, axes = framed);

```



```

> x0 := 1/2; y0 := -1/2;

```

$$x0 := \frac{1}{2}$$

$$y0 := \frac{-1}{2}$$

```

> f(x0, y0);

```

$$\frac{1}{4}\pi - \ln\left(\frac{3}{2}\right)$$

> D[1\$1, 2\$0](f)(x, y);

$$\frac{1}{1+(x-y)^2} - \frac{2x}{1+x^2+y^2}$$

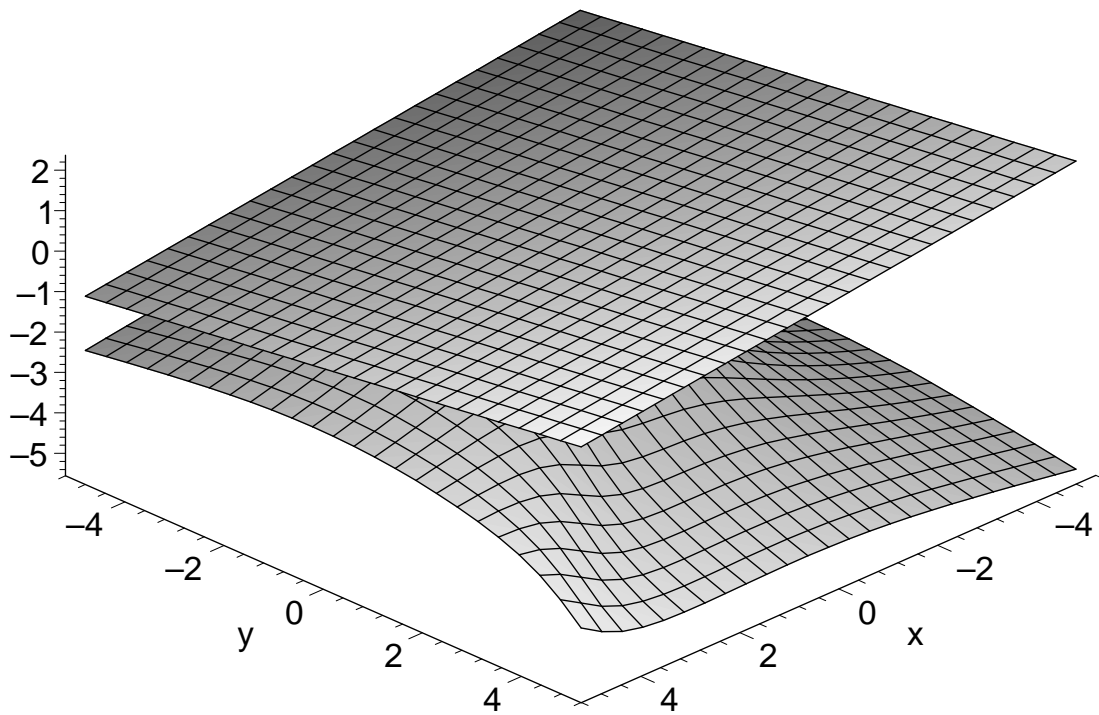
> D[1\$0, 2\$1](f)(x, y);

$$-\frac{1}{1+(x-y)^2} - \frac{2y}{1+x^2+y^2}$$

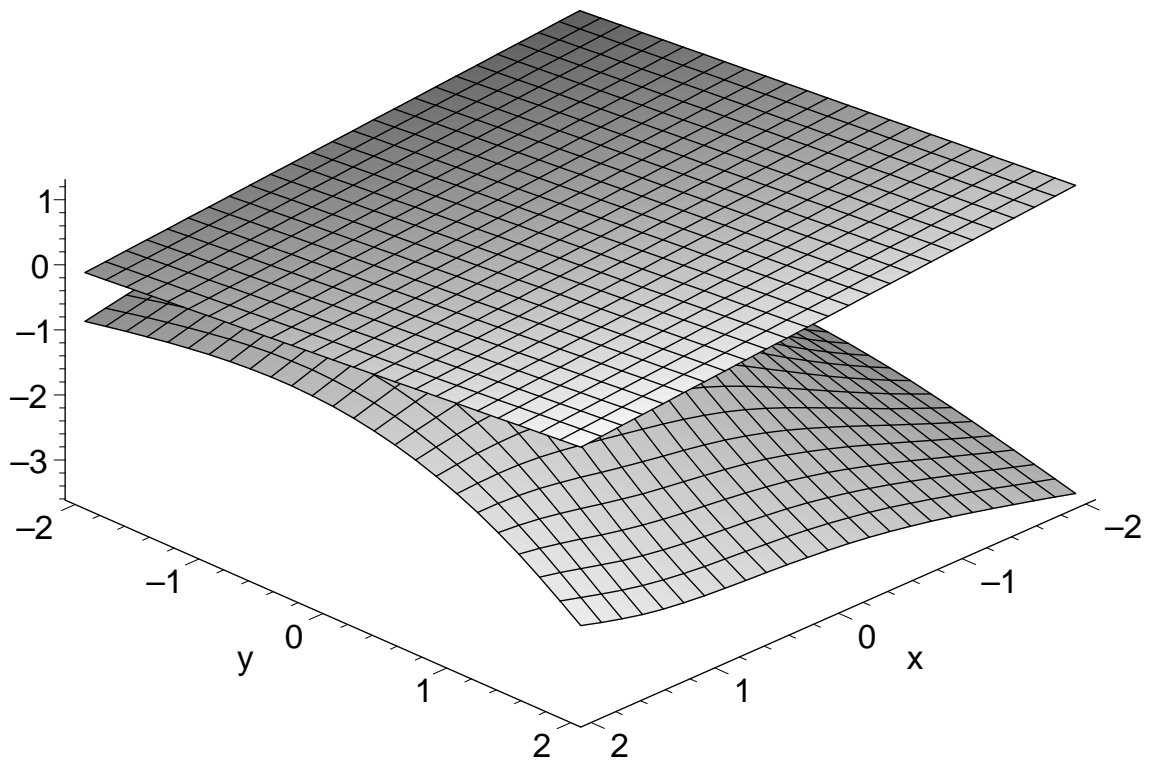
> T:=f(x0, y0)+D[1\$1, 2\$0](f)(x0, y0)*(x-x0)+D[1\$0, 2\$1](f)(x0, y0)*(y-y0);

$$T := \frac{1}{4}\pi - \ln\left(\frac{3}{2}\right) - \frac{1}{6}x + \frac{1}{6} + \frac{1}{6}y$$

> plot3d({T, f(x, y)}, x=-5..5, y=-5..5, axes=framed);



```
> plot3d({T,f(x,y)},x=-2..2,y=-2..2,axes=framed);
```



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[ >
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```
[ > f:=x->sin(x)+cos(2*x);
```

$$f := x \rightarrow \sin(x) + \cos(2x)$$

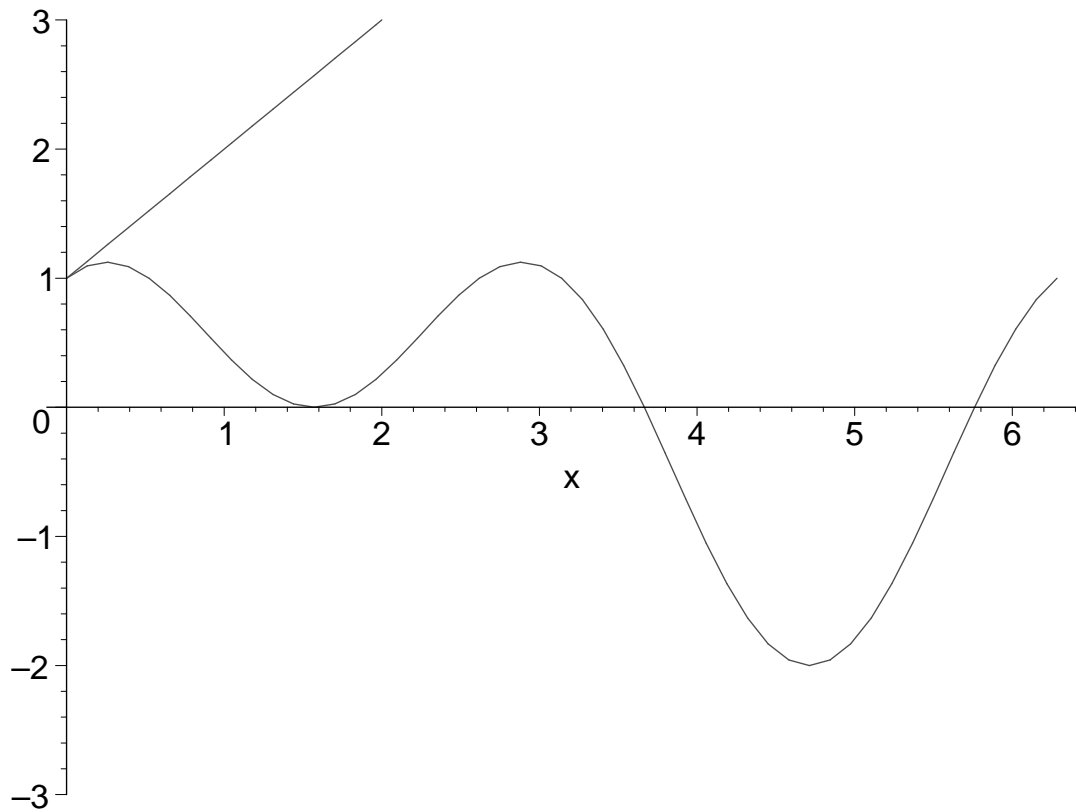
```
[ > t:=(x,a)->f(a)+D(f)(a)*(x-a);
```

$$t := (x, a) \rightarrow f(a) + D(f)(a)(x - a)$$

```
[ > with(plots):
```

```
Warning, the name changecoords has been redefined
```

```
[ > animate({f(x),t(x,s)},x=0..2*Pi,s=0..2*Pi,v  
view=-3..3,frames=100);
```



[>

[>

```
> a[0]:=0;a[1]:=1;b[0]:=1;b[1]:=1;for k from
0 to 20 do
a[k+2]:=(2*k+3)*a[k+1]+(k+1)^2*a[k];
b[k+2]:=(2*k+3)*b[k+1]+(k+1)^2*b[k];
c[k+2]:=4*a[k+2]/b[k+2];
od;
```

$$a_0 := 0$$

$$a_1 := 1$$

$$b_0 := 1$$

$$b_1 := 1$$

$$a_2 := 3$$

$$b_2 := 4$$

$$c_2 := 3$$

$$a_3 := 19$$

$$b_3 := 24$$

$$c_3 := \frac{19}{6}$$

$$a_4 := 160$$

$$b_4 := 204$$

$$c_4 := \frac{160}{51}$$

$$a_5 := 1744$$

$$b_5 := 2220$$

$$c_5 := \frac{1744}{555}$$

$$a_6 := 23184$$

$$b_6 := 29520$$

$$c_6 := \frac{644}{205}$$

$$a_7 := 364176$$

$$b_7 := 463680$$

$$c_7 := \frac{2529}{805}$$

$$a_8 := 6598656$$

$$b_8 := 8401680$$

$$c_8 := \frac{183296}{58345}$$

$$a_9 := 135484416$$

$$b_9 := 172504080$$

$$c_9 := \frac{3763456}{1197945}$$

$$a_{10} := 3108695040$$

$$b_{10} := 3958113600$$

$$c_{10} := \frac{4317632}{1374345}$$

$$a_{11} := 78831037440$$

$$b_{11} := 100370793600$$

$$c_{11} := \frac{54743776}{17425485}$$

$$a_{12} := 2189265960960$$

$$b_{12} := 2787459998400$$

$$c_{12} := \frac{1013549056}{322622685}$$

$$a_{13} := 66083318415360$$

$$b_{13} := 84139894238400$$

$$c_{13} := \frac{30594128896}{9738413685}$$

$$a_{14} := 2154235544616960$$

$$b_{14} := 2742857884166400$$

$$c_{14} := \frac{35618973952}{11337871545}$$

$$a_{15} := 75425161203302400$$

$$b_{15} := 96034297911552000$$

$$c_{15} := \frac{10392576224}{3308059755}$$

$$a_{16} := 2822882994841190400$$

$$b_{16} := 3594206259195552000$$

$$c_{16} := \frac{3111643512832}{990466892415}$$

$$a_{17} := 112463980097804697600$$

$$b_{17} := 143193586818810528000$$

$$c_{17} := \frac{123968232030208}{39460313827935}$$

$$a_{18} := 4752052488932268441600$$

$$b_{18} := 6050501147565883008000$$

$$c_{18} := \frac{48501417558016}{15438480702645}$$

$$a_{19} := 212264271642182654361600$$

$$b_{19} := 270263264589232282368000$$

$$c_{19} := \frac{1083228572868608}{344802363740835}$$

$$a_{20} := 9993797542549672427520000$$

$$b_{20} := 12724498233251342778240000$$

$$c_{20} := \frac{4080033616887808}{1298715036217599}$$

$$a_{21} := 494651407901409631272960000$$

$$b_{21} := 629809733398997966855040000$$

$$c_{21} := \frac{188557135970304}{60019600489849}$$

$$a_{22} := 25677275256025019685273600000$$

$$b_{22} := 32693322257020754739970560000$$

$$c_{22} := \frac{3781715948011520}{1203757572990973}$$

```
> evalf(seq(c[k], k=2..10));
```

```
3., 3.166666667, 3.137254902, 3.142342342, 3.141463415,
```

3.141614907, 3.141588825, 3.141593312, 3.141592540

```
> for k from 2 while evalf(abs(c[k]-Pi),15)>
(10^(-10))
do;print(k,c[k],evalf(abs(c[k]-Pi),15));
od;
```

2, 3, .14159265358979

3, $\frac{19}{6}$, .02507401307688

4, $\frac{160}{51}$, .00433775162901

5, $\frac{1744}{555}$, .00074968875255

6, $\frac{644}{205}$, .00012923895564

7, $\frac{2529}{805}$, .00002225324251

8, $\frac{183296}{58345}$, .382849767 10⁻⁵

9, $\frac{3763456}{1197945}$, .65829014 10⁻⁶

10, $\frac{4317632}{1374345}$, .11314325 10⁻⁶

11, $\frac{54743776}{17425485}$, .1944054 10⁻⁷

$$12, \frac{1013549056}{322622685}, .333954 \cdot 10^{-8}$$

$$13, \frac{30594128896}{9738413685}, .57358 \cdot 10^{-9}$$

[>
[>