```
> f:=x*y-x^3-y^2;
```

$$f := x\,y - x^3 - y^2$$

```
> fx:=diff(f,x);
```

$$fx := y - 3\,x^2$$

```
> fy:=diff(f,y);
```

$$fy := x - 2\,y$$

```
> s:=solve({fx,fy},{x,y});
```

$$s := \{y = 0, x = 0\}, \{y = \frac{1}{12}, x = \frac{1}{6}\}$$

```
> fxx:=diff(f,x,x);
```

$$fxx := -6\,x$$

```
> fyy:=diff(f,y,y);
```

$$fyy := -2$$

```
> fxy:=diff(f,x,y);
```

$$fxy := 1$$

```
> d:=fxx*fyy-fxy^2;
```

$$d := 12\,x - 1$$

```
> subs(s[1],d);
```

$$-1$$

```
> subs(s[2],d);
```

$$1$$

```
> s[2];
```

$$\{y = \frac{1}{12}, x = \frac{1}{6}\}$$

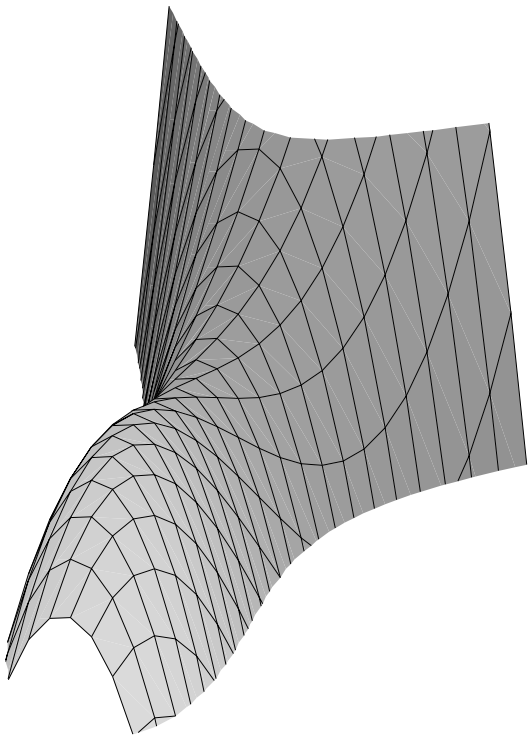
```
> subs(s[1],f);
```

$$0$$

```
> subs(s[2],f);
```

$$\frac{1}{432}$$

```
> plot3d(f,x=-0.3..0.3,y=-0.3..0.3,view=-0.01
  ..0.01);
```



```
> with(linalg);
Warning, new definition for norm
Warning, new definition for trace
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*,

augment, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*]

```
> f;
```

$$x\,y - x^3 - y^2$$

```
> grad(f,[x,y]);
```

$$[y - 3\,x^2,\ x - 2\,y]$$

```
> solve(%,{x,y});
Error, (in solve) invalid arguments
> convert(grad(f,[x,y]),set);
```

$$\{x - 2\,y,\ y - 3\,x^2\}$$

```
> solve(%,{x,y});
```

$$\{y = 0, x = 0\}, \{y = \frac{1}{12}, x = \frac{1}{6}\}$$

```
> hessian(f,[x,y]);
```

$$\begin{bmatrix} -6x & 1 \\ 1 & -2 \end{bmatrix}$$

```
> eigenvals(%);
```

$$-3x - 1 + \sqrt{9x^2 - 6x + 2}, -3x - 1 - \sqrt{9x^2 - 6x + 2}$$

```
>
> f:=(x^2-2*x*y+z^2)*exp(x-y-z^2);
```

$$f := (x^2 - 2xy + z^2)\, \mathbf{e}^{(x - y - z^2)}$$

```
> gr:=convert(grad(f,[x,y,z]),set);
```

$$gr := \{(2x - 2y)\, \mathbf{e}^{(x - y - z^2)} + (x^2 - 2xy + z^2)\, \mathbf{e}^{(x - y - z^2)},$$
$$-2x\, \mathbf{e}^{(x - y - z^2)} - (x^2 - 2xy + z^2)\, \mathbf{e}^{(x - y - z^2)},$$
$$2z\, \mathbf{e}^{(x - y - z^2)} - 2(x^2 - 2xy + z^2)\, z\, \mathbf{e}^{(x - y - z^2)}\}$$

```
> s:=solve(gr,{x,y,z});
```

$$s := \{z = 0, y = 0, x = 0\}, \{z = 0, y = 0, x = -2\},$$
$$\{x = \frac{-1}{2}, z = \frac{1}{2}\,\mathrm{RootOf}(-3 + \_Z^2), y = 0\}$$

```
> s[1];
```

$$\{z = 0, y = 0, x = 0\}$$

```
> s[2];
```

$$\{z = 0, y = 0, x = -2\}$$

```
> t:=allvalues(s[3]);
```

$$t := \{x = \frac{-1}{2}, z = \frac{1}{2}\sqrt{3}, y = 0\}, \{x = \frac{-1}{2}, z = -\frac{1}{2}\sqrt{3}, y = 0\}$$

```
> t[1];
```

$$\{x = \frac{-1}{2}, z = \frac{1}{2}\sqrt{3}, y = 0\}$$

```
> t[2];
```

$$\{x = \frac{-1}{2}, z = -\frac{1}{2}\sqrt{3}, y = 0\}$$

```
> he:=hessian(f,[x,y,z]);
```

$$he :=$$

$$[2\,\mathbf{e}^{(x-y-z^2)} + 2\,(2x-2y)\,\mathbf{e}^{(x-y-z^2)} + (x^2-2xy+z^2)\,\mathbf{e}^{(x-y-z^2)},$$

$$-2\,\mathbf{e}^{(x-y-z^2)} - 2x\,\mathbf{e}^{(x-y-z^2)} - (2x-2y)\,\mathbf{e}^{(x-y-z^2)}$$

$$-(x^2-2xy+z^2)\,\mathbf{e}^{(x-y-z^2)},\ 2z\,\mathbf{e}^{(x-y-z^2)}$$

$$-2\,(2x-2y)\,z\,\mathbf{e}^{(x-y-z^2)} - 2\,(x^2-2xy+z^2)\,z\,\mathbf{e}^{(x-y-z^2)}]$$

$$[-2\,\mathbf{e}^{(x-y-z^2)} - 2x\,\mathbf{e}^{(x-y-z^2)} - (2x-2y)\,\mathbf{e}^{(x-y-z^2)}$$

$$-(x^2-2xy+z^2)\,\mathbf{e}^{(x-y-z^2)},$$

$$4x\,\mathbf{e}^{(x-y-z^2)} + (x^2-2xy+z^2)\,\mathbf{e}^{(x-y-z^2)},$$

$$-2z\,\mathbf{e}^{(x-y-z^2)} + 4xz\,\mathbf{e}^{(x-y-z^2)} + 2\,(x^2-2xy+z^2)\,z\,\mathbf{e}^{(x-y-z^2)}]$$

$$[2z\,\mathbf{e}^{(x-y-z^2)} - 2\,(2x-2y)\,z\,\mathbf{e}^{(x-y-z^2)}$$

$$-2\,(x^2-2xy+z^2)\,z\,\mathbf{e}^{(x-y-z^2)},$$

$$-2\,z\,\mathbf{e}^{(x-y-z^2)} + 4\,x\,z\,\mathbf{e}^{(x-y-z^2)} + 2\,(x^2 - 2\,x\,y + z^2)\,z\,\mathbf{e}^{(x-y-z^2)},$$

$$2\,\mathbf{e}^{(x-y-z^2)} - 8\,\mathbf{e}^{(x-y-z^2)}\,z^2 - 2\,(x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)}$$

$$+ 4\,(x^2 - 2\,x\,y + z^2)\,z^2\,\mathbf{e}^{(x-y-z^2)}]$$

```
> he;
```

$$he$$

```
> matrix(he);
```

$$[2\,\mathbf{e}^{(x-y-z^2)} + 2\,(2\,x - 2\,y)\,\mathbf{e}^{(x-y-z^2)} + (x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)}$$

$$, -2\,\mathbf{e}^{(x-y-z^2)} - 2\,x\,\mathbf{e}^{(x-y-z^2)} - (2\,x - 2\,y)\,\mathbf{e}^{(x-y-z^2)}$$

$$- (x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)},\ 2\,z\,\mathbf{e}^{(x-y-z^2)}$$

$$- 2\,(2\,x - 2\,y)\,z\,\mathbf{e}^{(x-y-z^2)} - 2\,(x^2 - 2\,x\,y + z^2)\,z\,\mathbf{e}^{(x-y-z^2)}]$$

$$[-2\,\mathbf{e}^{(x-y-z^2)} - 2\,x\,\mathbf{e}^{(x-y-z^2)} - (2\,x - 2\,y)\,\mathbf{e}^{(x-y-z^2)}$$

$$- (x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)},$$

$$4\,x\,\mathbf{e}^{(x-y-z^2)} + (x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)},$$

$$-2\,z\,\mathbf{e}^{(x-y-z^2)} + 4\,x\,z\,\mathbf{e}^{(x-y-z^2)} + 2\,(x^2 - 2\,x\,y + z^2)\,z\,\mathbf{e}^{(x-y-z^2)}]$$

$$[2\,z\,\mathbf{e}^{(x-y-z^2)} - 2\,(2\,x - 2\,y)\,z\,\mathbf{e}^{(x-y-z^2)}$$

$$- 2\,(x^2 - 2\,x\,y + z^2)\,z\,\mathbf{e}^{(x-y-z^2)},$$

$$-2\,z\,\mathbf{e}^{(x-y-z^2)} + 4\,x\,z\,\mathbf{e}^{(x-y-z^2)} + 2\,(x^2 - 2\,x\,y + z^2)\,z\,\mathbf{e}^{(x-y-z^2)},$$

$$2\,\mathbf{e}^{(x-y-z^2)} - 8\,\mathbf{e}^{(x-y-z^2)}\,z^2 - 2\,(x^2 - 2\,x\,y + z^2)\,\mathbf{e}^{(x-y-z^2)}$$

$$+4\,(x^2-2\,x\,y+z^2)\,z^2\,\mathbf{e}^{(x-y-z^2)}]$$

```
> subs(s[1],matrix(he));
```

$$\begin{bmatrix} 2\,\mathbf{e}^{0} & -2\,\mathbf{e}^{0} & 0 \\ -2\,\mathbf{e}^{0} & 0 & 0 \\ 0 & 0 & 2\,\mathbf{e}^{0} \end{bmatrix}$$

```
> he1:=simplify(%);
```

$$he1 := \begin{bmatrix} 2 & -2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

```
> definite(he1,'negative_semidef');
```

$$false$$

```
> definite(he1,'positive_semidef');
```

$$false$$

```
> eigenvals(he1);
```

$$2,\ 1+\sqrt{5},\ 1-\sqrt{5}$$

```
> he2:=simplify(subs(s[2],matrix(he)));
```

$$he2 := \begin{bmatrix} -2\,\mathbf{e}^{(-2)} & 2\,\mathbf{e}^{(-2)} & 0 \\ 2\,\mathbf{e}^{(-2)} & -4\,\mathbf{e}^{(-2)} & 0 \\ 0 & 0 & -6\,\mathbf{e}^{(-2)} \end{bmatrix}$$

```
> definite(he2,'negative_def');
```

$$-2\,\mathbf{e}^{(-2)} < 0 \ \textbf{and}\ -4\,(\mathbf{e}^{(-2)})^2 < 0 \ \textbf{and}\ -24\,(\mathbf{e}^{(-2)})^3 < 0$$

```
> eigenvals(he2);
```

$$-6\,\mathbf{e}^{(-2)},\ -3\,\mathbf{e}^{(-2)}+\sqrt{5}\,\mathbf{e}^{(-2)},\ -3\,\mathbf{e}^{(-2)}-\sqrt{5}\,\mathbf{e}^{(-2)}$$

```
> evalf(%);
```

-.8120116992, -.1033869565, -.7086247427

```
> evalf(eigenvals(subs(t[1],matrix(he))));
```

$$.6887709964 - .1\ 10^{-9}\ I, -1.209530831 - .1\ 10^{-9}\ I, -.3387545559 + .1\ 10^{-9}\ I$$

```
> evalf(eigenvals(subs(t[2],matrix(he))));
```

$$.6887709964 - .1\ 10^{-9}\ I, -1.209530831 - .1\ 10^{-9}\ I, -.3387545559 + .1\ 10^{-9}\ I$$

```
> maximize(f,x=-3..3,y=-3..3,z=-3..3,location
  );
```

$$27\ \mathbf{e}^6, \{[\{x = 3, y = -3, z = 0\}, 27\ \mathbf{e}^6]\}$$

```
>
> A:=matrix(2,3,[2,1,1,3,4,5]);
```

$$A := \begin{bmatrix} 2 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

```
> b:=vector([6,7]);
```

$$b := [6, 7]$$

```
> linsolve(A,b);
```

$$[\_t_1, -7\ \_t_1 + 23, 5\ \_t_1 - 17]$$

```
> B:=matrix(2,2,[[1,2],[u,v]]);
```

$$B := \begin{bmatrix} 1 & 2 \\ u & v \end{bmatrix}$$

```
> C:=multiply(B,A);
```

$$C := \begin{bmatrix} 8 & 9 & 11 \\ 2\ u + 3\ v & u + 4\ v & u + 5\ v \end{bmatrix}$$

```
> multiply(A,B);
Error, (in multiply) non matching dimensions for
```

vector/matrix product

```
> multiply(B,B,A);
```

$$\begin{bmatrix} 8+4u+6v & 9+2u+8v & 11+2u+10v \\ 8u+2vu+3v^2 & 9u+vu+4v^2 & 11u+vu+5v^2 \end{bmatrix}$$

```
> transpose(C);
```

$$\begin{bmatrix} 8 & 2u+3v \\ 9 & u+4v \\ 11 & u+5v \end{bmatrix}$$

```
> C;
```

$$C$$

```
> print(C);
```

$$\begin{bmatrix} 8 & 9 & 11 \\ 2u+3v & u+4v & u+5v \end{bmatrix}$$

```
> matrix(B);
```

$$\begin{bmatrix} 1 & 2 \\ u & v \end{bmatrix}$$

```
> det(B);
```

$$v-2u$$

```
> inverse(B);
```

$$\begin{bmatrix} -\frac{v}{-v+2u} & 2\frac{1}{-v+2u} \\ \frac{u}{-v+2u} & -\frac{1}{-v+2u} \end{bmatrix}$$

```
> charpoly(B,x);
```

$$x^2-xv-x+v-2u$$

```
> factor(%);
```

$$x^2 - x\,v - x + v - 2\,u$$

```
> eigenvals(B);
```

$$\frac{1}{2}+\frac{1}{2}v+\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u},\ \frac{1}{2}+\frac{1}{2}v-\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u}$$

```
> eigenvectors(B);
```

$$\left[\frac{1}{2}+\frac{1}{2}v+\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u},\ 1,\ \left\{\left[\begin{array}{c}-\dfrac{-\frac{1}{2}+\frac{1}{2}v-\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u}}{u},\ 1\end{array}\right]\right\}\right],\left[\frac{1}{2}+\frac{1}{2}v-\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u},\ 1,\ \left\{\left[\begin{array}{c}-\dfrac{-\frac{1}{2}+\frac{1}{2}v+\frac{1}{2}\sqrt{1-2\,v+v^2+8\,u}}{u},\ 1\end{array}\right]\right\}\right]$$

```
> B0:=subs({u=1,v=2},matrix(B));
```

$$B0 := \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

```
> eigenvals(B0);
```

$$0,\ 3$$

```
> eigenvectors(B0);
```

$$[3,\ 1,\ \{[1,\ 1]\}],\ [0,\ 1,\ \{[-2,\ 1]\}]$$

```
> F:=(i,j)->1/(i+j);
```

$$F := (i, j) \to \frac{1}{i + j}$$

```
> F(3,4);
```

$$\frac{1}{7}$$

```
> M:=matrix(5,5,F);
```

$$M := \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \end{bmatrix}$$

```
> det(M);
```

$$\frac{1}{67212633600000}$$

```
>
```