```
> f:=sqrt(x^2+y^2)*(y^3-5*y+x);
```

$$f := \sqrt{x^2+y^2}\,(y^3-5\,y+x)$$

```
> with(linalg):
>
> gr:=convert(grad(f,[x,y]),set);
```

$$gr := \{\frac{(y^3-5\,y+x)\,y}{\sqrt{x^2+y^2}}+\sqrt{x^2+y^2}\,(3\,y^2-5),\ \frac{(y^3-5\,y+x)\,x}{\sqrt{x^2+y^2}}+\sqrt{x^2+y^2}\}$$

```
> s:=solve(gr,{x,y});
```

$$s := \{x = \text{RootOf}(\_Z^2-2),\ y = \text{RootOf}(\_Z^2-2)\},$$
$$\{y = \text{RootOf}(6\,\_Z^2-13),\ x = \frac{2}{3}\,\text{RootOf}(6\,\_Z^2-13)\}$$

```
> s1:=allvalues(s[1]);
```

$$s1 := \{x=\sqrt{2},\ y=\sqrt{2}\},\ \{x=-\sqrt{2},\ y=-\sqrt{2}\}$$

```
> s2:=allvalues(s[2]);
```

$$s2 := \{x=\frac{1}{9}\sqrt{78},\ y=\frac{1}{6}\sqrt{78}\},\ \{y=-\frac{1}{6}\sqrt{78},\ x=-\frac{1}{9}\sqrt{78}\}$$

```
> evalf(%);
```

$$\{x = .9813067628,\ y = 1.471960145\},\ \{x = -.9813067628,\ y = -1.471960145\}$$

```
> he:=hessian(f,[x,y]);
```

$$he :=$$
$$\left[-\frac{(y^3-5\,y+x)\,x^2}{(x^2+y^2)^{(3/2)}}+2\,\frac{x}{\sqrt{x^2+y^2}}+\frac{y^3-5\,y+x}{\sqrt{x^2+y^2}},\ -\frac{(y^3-5\,y+x)\,y\,x}{(x^2+y^2)^{(3/2)}}+\frac{y}{\sqrt{x^2+y^2}}+\frac{(3\,y^2-5)\,x}{\sqrt{x^2+y^2}}\right]$$
$$\left[-\frac{(y^3-5\,y+x)\,y\,x}{(x^2+y^2)^{(3/2)}}+\frac{y}{\sqrt{x^2+y^2}}+\frac{(3\,y^2-5)\,x}{\sqrt{x^2+y^2}},\right.$$
$$\left.-\frac{(y^3-5\,y+x)\,y^2}{(x^2+y^2)^{(3/2)}}+2\,\frac{(3\,y^2-5)\,y}{\sqrt{x^2+y^2}}+\frac{y^3-5\,y+x}{\sqrt{x^2+y^2}}+6\sqrt{x^2+y^2}\,y\right]$$

```
> subs(s1[1],matrix(he));
```

$$\begin{bmatrix}\frac{1}{4}\sqrt{4}\,\sqrt{2} & \frac{3}{4}\sqrt{4}\,\sqrt{2}\\ \frac{3}{4}\sqrt{4}\,\sqrt{2} & \frac{25}{4}\sqrt{4}\,\sqrt{2}\end{bmatrix}$$

```
> eigenvals(%);
```

$$\frac{13}{2}\sqrt{2}+\frac{3}{2}\sqrt{34},\ \frac{13}{2}\sqrt{2}-\frac{3}{2}\sqrt{34}$$

```
> evalf(%);
```

$$17.93881600, .445960310$$

```
> eigenvals(subs(s1[2],matrix(he)));
```

$$-\frac{13}{2}\sqrt{2}+\frac{3}{2}\sqrt{34}, -\frac{13}{2}\sqrt{2}-\frac{3}{2}\sqrt{34}$$

```
> evalf(%);
```

$$-.445960310, -17.93881600$$

```
> eigenvals(subs(s2[1],matrix(he)));
```

$$\frac{29}{12}\sqrt{13}+\frac{1}{12}\sqrt{12181}, \frac{29}{12}\sqrt{13}-\frac{1}{12}\sqrt{12181}$$

```
> evalf(%);
```

$$17.91071289, -.483881726$$

```
> eigenvals(subs(s2[2],matrix(he)));
```

$$-\frac{29}{12}\sqrt{13}+\frac{1}{12}\sqrt{12181}, -\frac{29}{12}\sqrt{13}-\frac{1}{12}\sqrt{12181}$$

```
> evalf(%);
```

$$.483881726, -17.91071289$$

```
> subs(s1[1],f);
```

$$-2\sqrt{4}\sqrt{2}$$

```
> simplify(subs(s1[2],f));
```

$$4\sqrt{2}$$

```
> evalf(%);
```

$$5.656854248$$

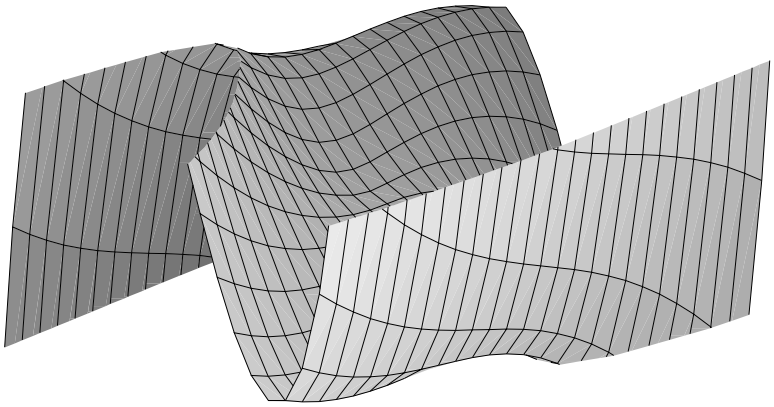
```
> simplify(subs(s2[2],f));
```

$$\frac{169}{108}\sqrt{13}$$

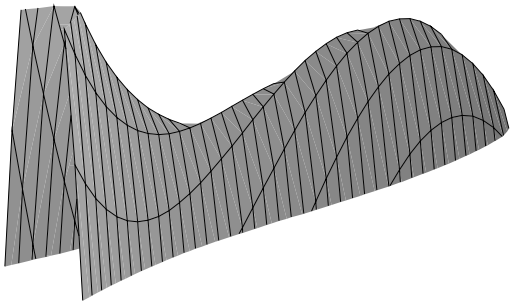
```
> evalf(%);
```

$$5.642020051$$

```
> plot3d(f,x=-3..3,y=-3..3,view=-8..8);
```



```
> plot3d(f,x=-1.8..-0.5,y=-1.8..-0.5,view=5.6..5.7,grid=[40,40]);
```



```
> minimize(f,x=-2..2,y=-2..2,location);
```

$$\sqrt{4+\text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)^2}\ (\text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)^3-5\ \text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)-2),\ \{[\{y=\text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1),\ x=-2\},\ \sqrt{4+\text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)^2}\ (\text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)^3-5\ \text{RootOf}(2\_Z^4+\_Z^2-\_Z-10,\ index=1)-2)]\}$$

```
> evalf(%);
```

$$-15.32329940,\ \{[\{x=-2.,\ y=1.468906493\},\ -15.32329940]\}$$

```
> f:=2*x^4+y^4+z^4-4*x*y*z-2*x^2;
```

$$f := 2x^4+y^4+z^4-4xyz-2x^2$$

```
> gr:=convert(grad(f,[x,y,z]),set);
```

$$gr := \{8x^3-4yz-4x,\ 4y^3-4xz,\ 4z^3-4xy\}$$

```
> s:=solve(gr,{x,y,z});
```

$$s := \{y=0,x=0,z=0\},\ \{y=0,z=0,x=\text{RootOf}(2\_Z^2-1)\},\ \{x=1,y=1,z=1\},\ \{x=1,y=-1,z=-1\},\ \{x=-1,y=\text{RootOf}(\_Z^2+1),z=\text{RootOf}(\_Z^2+1)\},\ \{y=-1,x=-1,z=1\},\ \{y=1,x=-1,z=-1\},\ \{x=1,y=-\text{RootOf}(\_Z^2+1),z=\text{RootOf}(\_Z^2+1)\}$$

```
> s[1];
```

$$\{y=0,x=0,z=0\}$$

```
> s2:=allvalues(s[2]);
```

$$s2 := \{x=\frac{1}{2}\sqrt{2},y=0,z=0\},\ \{x=-\frac{1}{2}\sqrt{2},y=0,z=0\}$$

```
> s[3];
```

$$\{x=1,y=1,z=1\}$$

```
> s[4];
```

$$\{x=1,y=-1,z=-1\}$$

```
> allvalues(s[5]);
```

$$\{x=-1,z=I,y=I\},\ \{x=-1,y=-I,z=-I\}$$

```
> s[6];
```

$$\{y=-1,x=-1,z=1\}$$

```
> s[7];
```

$$\{y=1,x=-1,z=-1\}$$

```
> he:=hessian(f,[x,y,z]);
```

$$he := \begin{bmatrix} 24x^2-4 & -4z & -4y \\ -4z & 12y^2 & -4x \\ -4y & -4x & 12z^2 \end{bmatrix}$$

```
> subs(s[1],matrix(he));
```

$$\begin{bmatrix} -4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> subs(s2[1],matrix(he));
```

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & -2\sqrt{2} \\ 0 & -2\sqrt{2} & 0 \end{bmatrix}$$

```
> eigenvals(%);
```

$$8, 2\sqrt{2}, -2\sqrt{2}$$

```
> subs(s2[2],matrix(he));
```

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 0 & 2\sqrt{2} \\ 0 & 2\sqrt{2} & 0 \end{bmatrix}$$

```
> eigenvals(%);
```

$$8, 2\sqrt{2}, -2\sqrt{2}$$

```
> subs(s[3],matrix(he));
```

$$\begin{bmatrix} 20 & -4 & -4 \\ -4 & 12 & -4 \\ -4 & -4 & 12 \end{bmatrix}$$

```
> eigenvals(%);
```

$$16, 14 + 2\sqrt{17}, 14 - 2\sqrt{17}$$

```
> evalf(%);
```

$$16., 22.24621125, 5.753788748$$

```
> subs(s[4],matrix(he));
```

$$\begin{bmatrix} 20 & 4 & 4 \\ 4 & 12 & -4 \\ 4 & -4 & 12 \end{bmatrix}$$

```
> eigenvals(%);
```

$$16, 14 + 2\sqrt{17}, 14 - 2\sqrt{17}$$

```
> subs(s[6],matrix(he));
```

$$\begin{bmatrix} 20 & -4 & 4 \\ -4 & 12 & 4 \\ 4 & 4 & 12 \end{bmatrix}$$

```
> eigenvals(%);
```

$$16, 14 + 2\sqrt{17}, 14 - 2\sqrt{17}$$

```
> subs(s[7],matrix(he));
```

$$\begin{bmatrix} 20 & 4 & -4 \\ 4 & 12 & 4 \\ -4 & 4 & 12 \end{bmatrix}$$

```
> eigenvals(%);
```

$$16,\ 14 + 2\sqrt{17},\ 14 - 2\sqrt{17}$$

```
> subs(s[7],f);
```

$$-2$$

```
> minimize(f,x=-infinity..infinity,y=-infinity..infinity,z=-infinity
  ..infinity,location);
```

$$\min(-2,\ 2\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^2,$$
$$2\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^2),\ \{[\{x=-1,\ z=-1,\ y=1\},\ -2],$$
$$[\{z=1,\ x=1,\ y=1\},\ -2],\ [\{x=1,\ y=-1,\ z=-1\},\ -2],\ [\{z=1,\ x=-1,\ y=-1\},\ -2],\ [$$
$$\{x=-1,\ y=\mathrm{RootOf}(\_Z^2+1,\ index=1),\ z=\mathrm{RootOf}(\_Z^2+1,\ index=1)\},$$
$$2\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^2],\ [$$
$$\{x=-1,\ z=\mathrm{RootOf}(\_Z^2+1,\ index=2),\ y=\mathrm{RootOf}(\_Z^2+1,\ index=2)\},$$
$$2\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^2],\ [$$
$$\{x=1,\ y=-\mathrm{RootOf}(\_Z^2+1,\ index=1),\ z=\mathrm{RootOf}(\_Z^2+1,\ index=1)\},$$
$$2\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=1)^2],\ [$$
$$\{x=1,\ z=\mathrm{RootOf}(\_Z^2+1,\ index=2),\ y=-\mathrm{RootOf}(\_Z^2+1,\ index=2)\},$$
$$2\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^4 + 4\,\mathrm{RootOf}(\_Z^2+1,\ index=2)^2]\}$$

```
> A:=matrix(3,3,[1,2,3,4,5,4,3,2,1]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

```
> det(A);
```

$$-8$$

```
> ei:=eigenvals(A);
```

$$ei := -2,\ \frac{9}{2}+\frac{1}{2}\sqrt{65},\ \frac{9}{2}-\frac{1}{2}\sqrt{65}$$

```
> eigenvectors(A);
```

$$\left[\frac{9}{2}+\frac{1}{2}\sqrt{65},\ 1,\ \left\{\left[1,\ \frac{1}{4}+\frac{1}{4}\sqrt{65},\ 1\right]\right\}\right],\left[\frac{9}{2}-\frac{1}{2}\sqrt{65},\ 1,\ \left\{\left[1,\ \frac{1}{4}-\frac{1}{4}\sqrt{65},\ 1\right]\right\}\right],\ [-2,\ 1,\ \{[-1,\ 0,\ 1]\}]$$

```
> charpoly(A,x);
```

$$x^3 - 7x^2 - 14x + 8$$

```
> 
> inverse(A);
```

$$\begin{bmatrix} \frac{3}{8} & \frac{-1}{2} & \frac{7}{8} \\ -1 & 1 & -1 \\ \frac{7}{8} & \frac{-1}{2} & \frac{3}{8} \end{bmatrix}$$

```
> multiply(A,A);
```

$$\begin{bmatrix} 18 & 18 & 14 \\ 36 & 41 & 36 \\ 14 & 18 & 18 \end{bmatrix}$$

```
> multiply(A,A,A);
```

$$\begin{bmatrix} 132 & 154 & 140 \\ 308 & 349 & 308 \\ 140 & 154 & 132 \end{bmatrix}$$

```
> b:=matrix(3,1,[1,2,c]);
```

$$b := \begin{bmatrix} 1 \\ 2 \\ c \end{bmatrix}$$

```
> linsolve(A,b);
```

$$\begin{bmatrix} -\frac{5}{8} + \frac{7}{8} c \\ 1 - c \\ -\frac{1}{8} + \frac{3}{8} c \end{bmatrix}$$

```
> A:=matrix(3,3,[1,2,3,4,a,4,3,2,1]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & a & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

```
> det(A);
```

$$-8\,a + 32$$

```
> eigenvals(A);
```

$$-2,\ 2 + \frac{1}{2} a + \frac{1}{2} \sqrt{80 - 8\,a + a^2},\ 2 + \frac{1}{2} a - \frac{1}{2} \sqrt{80 - 8\,a + a^2}$$

```
> eigenvectors(A);
```

$$[-2,\ 1,\ \{[-1,\ 0,\ 1]\}],\left[2 + \frac{1}{2} a + \frac{1}{2} \sqrt{80 - 8\,a + a^2},\ 1,\ \left\{\left[1,\ -1 + \frac{1}{4} a + \frac{1}{4} \sqrt{80 - 8\,a + a^2},\ 1\right]\right\}\right],$$

$$\left[2+\frac{1}{2}a-\frac{1}{2}\sqrt{80-8a+a^2},\ 1,\ \left\{\left[1,\ -1+\frac{1}{4}a-\frac{1}{4}\sqrt{80-8a+a^2},\ 1\right]\right\}\right]$$

```
> charpoly(A,x);
```

$$x^3-2x^2-ax^2+2ax-24x+8a-32$$

```
>
> inverse(A);
```

$$\begin{bmatrix} -\frac{1}{8}\frac{a-8}{a-4} & -\frac{1}{2}\frac{1}{a-4} & \frac{1}{8}\frac{-8+3a}{a-4} \\ -\frac{1}{a-4} & \frac{1}{a-4} & -\frac{1}{a-4} \\ \frac{1}{8}\frac{-8+3a}{a-4} & -\frac{1}{2}\frac{1}{a-4} & -\frac{1}{8}\frac{a-8}{a-4} \end{bmatrix}$$

```
> multiply(A,A);
```

$$\begin{bmatrix} 18 & 8+2a & 14 \\ 16+4a & 16+a^2 & 16+4a \\ 14 & 8+2a & 18 \end{bmatrix}$$

```
> multiply(A,A,A);
```

$$\begin{bmatrix} 92+8a & 64+(8+2a)a & 100+8a \\ 128+16a+4a^2 & 64+16a+(16+a^2)a & 128+16a+4a^2 \\ 100+8a & 64+(8+2a)a & 92+8a \end{bmatrix}$$

```
> linsolve(A,b);
```

$$\begin{bmatrix} \frac{1}{8}\frac{-8c-a+3ac}{a-4} \\ -\frac{-1+c}{a-4} \\ -\frac{1}{8}\frac{-3a+ac+16-8c}{a-4} \end{bmatrix}$$

```
> subs(a=4,%);
Error, division by zero
> A0:=subs(a=4,matrix(A));
```

$$A0:=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

```
> linsolve(A0,b);
> eigenvals(A0);
```

$$0,\ -2,\ 8$$

```
>
> A1:=subs(a=2/3,matrix(A));
```

$$A1 := \begin{bmatrix} 1 & 2 & 3 \\ 4 & \frac{2}{3} & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

```
> eigenvals(A1);
```

$$\frac{20}{3}, -2, -2$$

```
> eigenvectors(A1);
```

$$\left[\frac{20}{3}, 1, \left\{\left[1, \frac{4}{3}, 1\right]\right\}\right], \left[-2, 2, \left\{\left[0, \frac{-3}{2}, 1\right], \left[1, \frac{-3}{2}, 0\right]\right\}\right]$$

```
>
```