COMMENTS AND CORRECTIONS TO "ITERATION OF MEROMORPHIC FUNCTIONS"

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These comments and corrections concern my survey paper [2]. They are updated when I receive new relevant information.

COMMENTS

Question 2: A more elementary proof of Theorem 4 has now been given by W. Schwick [20] for entire and and rational functions; his proof has been further simplified by D. Bargmann [1] and by F. Berteloot and J. Duval [6]. A. Bolsch [7] has extended Schwick's proof to meromorphic functions. In fact, he has generalized the result to functions with a countable compact set of essential singularities.

Question 3: M. Herring [14] has shown that the answer is "no". A. Bolsch has pointed out that this also follows from a result of M. Heins [13, Theorem 4]. A. Bolsch [8, 9] has generalized the result to functions with a countable compact set of essential singularities.

Question 4: In some sense the answer is "no". The function $f(z) = 2 - \log 2 + 2z - e^z$ has a Baker domain U such that the Euclidean distance between $\bigcup_{n=0}^{\infty} f^n(\mathrm{sing}(f^{-1}))$ and U is positive; see [3]. L. Rempe's answer [16] to Question 5 provides another example relevant to this question.

Question 5: L. Rempe [16] has shown that the answer is "yes".

Question 6: A. Bolsch [8, 9] has shown that the answer is "yes".

Question 7: M. Kisaka and M. Shishikura [15] have shown that the answer is "yes".

Question 10: M. Kisaka and M. Shishikura [15] have shown that the answer is "yes".

Question 14: C.-L. Cao and Y.-F. Wang [10] and, independently, W. Bergweiler and A. Eremenko [4] have shown that the answer is "yes" if $f \in S$.

Question 15: P. J. Rippon and G. M. Stallard [18] have shown that $I(f)$ has *at least one* unbounded component.

Question 16: Rottenfußer, Rückert, Rempe and Schleicher [19] have shown that the answer is "no".

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page 164, line 7: Z.-M. Gong, W.-Y. Qiu and F.-Y. Ren [12] and P. Domínguez [11, Theorem C] have shown that functions in the class P do not have Herman rings.

page 172, lines 5-7: W. Bergweiler and N. Terglane [5] have proved that the conclusion of Theorem 14 remains valid for $f \in R$, and in fact for a more general class of functions.

CORRECTIONS

page 166: the first formula in the proof should read:

$$[g^n(z), g^n(z')]_\Omega \leq [g^n(z), g^n(z')]_G \leq [g^n(z), g^n(z')]_{g^n(D)} \leq [z, z']_D$$

page 171, line 5: the reference for Fatou's example is [72], not [71]

page 173: P. J. Rippon and G. M. Stallard [17] have pointed out that Lemma 8 is false and that

$$f(z) = \frac{\tan z}{z} + \frac{\pi}{2}$$

is a counterexample (for $p = 2$). They show, however, that Theorem 16, in the proof of which Lemma 8 is used, is still correct. In fact, they prove [17, Theorem A] a generalization of Theorem 16.

To see that the conclusion of Theorem 16 holds it suffices to show that a path Γ with the property described in the conclusion of Lemma 8 exists if ∞ is not in the derived set of $\bigcup_{j=0}^{p-1} f^j(\text{sing}(f^{-1}))$. This follows from [17, Lemma 2.1].

page 176: the end of the first paragraph should read:

Hence $O^+(C) \subset C$. This is a contradiction, because $J \backslash O^+(C)$ contains at most the exceptional points of f and hence $O^+(C)$ is unbounded.

page 177, lines 2-3: replace "if an entire function f does not have Baker domains" by "if the Fatou set of an entire function f does not have components where f^n tends to ∞."

page 178, line 10: It should be

$$S_j = \{z : \text{Re}\, z > 1, (2j - 1)\pi < \text{Im}\, z < (2j + 1)\pi\}.$$

page 183: Reference [25] is by I. N. Baker and P. J. Rippon.

REFERENCES

[1] D. Bargmann, Simple proofs of some fundamental properties of the Julia set, *Ergodic Theory Dynamical Systems* 19 (1999), 553–558.

[2] W. Bergweiler, Iteration of meromorphic functions, *Bull. Amer. Math. Soc. (N. S.)* 29 (1993), 151–188.

[3] ———, Invariant domains and singularities, *Math. Proc. Cambridge Philos. Soc.* 117 (1995), 525–532.

[4] W. Bergweiler and A. Eremenko, Meromorphic functions with two completely invariant domains, in Transcendental Dynamics and Complex Analysis, edited by P. J. Rippon and G. M. Stallard, Cambridge University Press, LMS Lecture Note Series 348 (2008), 74–89.

[5] W. Bergweiler and N. Terglane, On the zeros of solutions of linear differential equations of the second order, *J. London Math. Soc.* 58 (1998), 311–330.

[6] F. Berteloot and J. Duval, Une démonstration directe de la densité des cycles répulsifs dans l'ensemble de Julia, in: Dolbeault, P. (ed.) et al., *Complex analysis and geometry.* Basel: Birkhaeuser. *Prog. Math.* 188, 221–222 (2000).

[7] A. Bolsch, Repulsive periodic points of meromorphic functions. *Complex Variables Theory Appl.* 31 (1996), 75–79.

[8] ———, *Iteration of meromorphic functions with countably many essential singularities*, Dissertation, Berlin 1997.

[9] ———, Periodic Fatou components of meromorphic functions, *Bull. London Math. Soc.* 31 (1999), 543–555.

[10] C.-L. Cao and Y.-F. Wang, On completely invariant Fatou components, *Ark. Mat.* 41 (2003), 253–265.

[11] P. Domínguez, Dynamics of transcendental meromorphic functions, *Ann. Acad. Sci. Fenn. Math.* 23 (1998), 225–250.

[12] Z.-M. Gong, W.-Y. Qiu and F.-Y. Ren A negative answer to a problem of Bergweiler, *Complex Variables Theory Appl.* 30 (1996), 315–322.

[13] M. Heins, Asymptotic spots of entire and meromorphic functions, *Ann. of Math.* 66 (1957), 430–439.

[14] M. Herring, Mapping properties of Fatou components, *Ann. Acad. Sci. Fenn. Math.* 23 (1998), 263–274.

[15] M. Kisaka and M. Shishikura, On multiply connected wandering domains of entire functions, in Transcendental Dynamics and Complex Analysis, edited by P. J. Rippon and G. M. Stallard, Cambridge University Press, LMS Lecture Note Series 348 (2008), 217–250.

[16] L. Rempe, Singular orbits and Baker domains, arXiv:2009.07020.

[17] P. J. Rippon and G. M. Stallard, Iteration of a class of hyperbolic meromorphic functions, *Proc. Amer. Math. Soc.* 127 (1999), 3251–3258.

[18] ———, On questions of Fatou and Eremenko, *Proc. Amer. Math. Soc.* 133 (2005), 1119–1126

[19] G. Rottenfußer, J. Rückert, L. Rempe and D. Schleicher, Dynamic rays of bounded-type entire functions, *Ann. of Math.* (2) 173 (2011), 77-125.

[20] W. Schwick, Repelling periodic points in the Julia set, *Bull. London Math. Soc.* 29 (1997), 314–316.