COMMENTS AND CORRECTIONS TO "ITERATION OF MEROMORPHIC FUNCTIONS"

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These comments and corrections concern my survey paper [2]. They are updated when I receive new relevant information.

Comments

Question 2: A more elementary proof of Theorem 4 has now been given by W. Schwick [19] for entire and and rational functions; his proof has been further simplified by D. Bargmann [1] and by F. Berteloot and J. Duval [6]. A. Bolsch [7] has extended Schwick's proof to meromorphic functions. In fact, he has generalized the result to functions with a countable compact set of essential singularities.

Question 3: M. Herring [14] has shown that the answer is "no". A. Bolsch has pointed out that this also follows from a result of M. Heins [13, Theorem 4]. A. Bolsch [8, 9] has generalized the result to functions with a countable compact set of essential singularities.

Question 4: In some sense the answer is "no". The function $f(z) = 2 - \log 2 + 2z - e^z$ has a Baker domain U such that the Euclidean distance between $\bigcup_{n=0}^{\infty} f^n(\text{sing}(f^{-1}))$ and U is positive; see [3].

Question 6: A. Bolsch [8, 9] has shown that the answer is "yes".

Question 7: M. Kisaka and M. Shishikura [15] have shown that the answer is "yes".

Question 10: M. Kisaka and M. Shishikura [15] have shown that the answer is "yes".

Question 14: C.-L. Cao and Y.-F. Wang [10] and, independently, W. Bergweiler and A. Eremenko [4] have shown that the answer is "yes" if $f \in S$.

Question 15: P. J. Rippon and G. M. Stallard [17] have shown that $I(f)$ has *at least one* unbounded component.

Question 16: Rottenfußer, Rückert, Rempe and Schleicher [18] have shown that the answer is "no".

page 164, line 7: Z.-M. Gong, W.-Y. Qiu and F.-Y. Ren [12] and P. Domínguez [11, Theorem C] have shown that functions in the class P do not have Herman rings.

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page 172, lines 5-7: W. Bergweiler and N. Terglane [5] have proved that the conclusion of Theorem 14 remains valid for $f \in R$, and in fact for a more general class of functions.

Corrections

page 166: the first formula in the proof should read:

$$[g^n(z), g^n(z')]_\Omega \leq [g^n(z), g^n(z')]_G \leq [g^n(z), g^n(z')]_{g^n(D)} \leq [z, z']_D$$

page 171, line 5: the reference for Fatou's example is [72], not [71]

page 173: P. J. Rippon and G. M. Stallard [16] have pointed out that Lemma 8 is false and that

$$f(z) = \frac{\tan z}{z} + \frac{\pi}{2}$$

is a counterexample (for $p = 2$). They show, however, that Theorem 16, in the proof of which Lemma 8 is used, is still correct. In fact, they prove [16, Theorem A] a generalization of Theorem 16.

To see that the conclusion of Theorem 16 holds it suffices to show that a path Γ with the property described in the conclusion of Lemma 8 exists if ∞ is not in the derived set of $\bigcup_{j=0}^{p-1} f^j(\text{sing}(f^{-1}))$. This follows from [16, Lemma 2.1].

page 176: the end of the first paragraph should read:

Hence $O^+(C) \subset C$. This is a contradiction, because $J \backslash O^+(C)$ contains at most the exceptional points of f and hence $O^+(C)$ is unbounded.

page 177, lines 2-3: replace "if an entire function f does not have Baker domains" by "if the Fatou set of an entire function f does not have components where f^n tends to ∞."

page 178, line 10: It should be

$$S_j = \{z : \text{Re}\, z > 1, (2j-1)\pi < \text{Im}\, z < (2j+1)\pi\}.$$

page 183: Reference [25] is by I. N. Baker and P. J. Rippon.

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