

# CORRECTION TO “MEROMORPHIC SOLUTIONS OF SOME FUNCTIONAL EQUATIONS”

WALTER BERGWEILER, KATSUYA ISHIZAKI AND NIRO YANAGIHARA

In [1] we treated the functional equation

$$(1.1) \quad \sum_{j=0}^n a_j(z) f(c^j z) = Q(z),$$

where  $0 < |c| < 1$  is a complex number, and  $a_j(z)$ ,  $j = 0, 1, \dots, n$ , and  $Q(z)$  are rational functions,  $a_0(z) \not\equiv 0$ ,  $a_n(z) \equiv 1$ . We mentioned that *if all coefficients of (1.1) are constant, then (1.1) has no transcendental meromorphic solution*. The argument given in [1], however, is not correct. The purpose of this note is to give a correct proof of this claim.

To do this we assume that (1.1) with constant coefficients  $a_j$  and  $Q$  possesses a meromorphic solution  $f$ . First we note that if  $f$  has a pole  $z_0 \neq 0$ , then, by (1.1),  $f$  has infinitely many poles of the form  $c^j z_0$ ,  $j \in \mathbb{N}$ , so that the poles of  $f$  accumulate at 0, a contradiction. Thus the only possible pole of  $f$  is at 0.

We now argue analogously to the proof of Theorem 1.1 in [1]. As there we put  $s = 1/c$  and we define

$$M_k = \max_{j=0,1,\dots,k} M(|s|^j, f) + 1$$

and  $L_k = \log M_k$  for  $k \geq 0$ . It follows from (1.1) that

$$|a_0| M(|s|^k, f) \leq \sum_{j=1}^n |a_j| M(|s|^{k-j}, f) + |Q| \leq \left( \sum_{j=1}^n |a_j| \right) M_{k-1} + |Q|$$

for  $k \geq n$ , and this implies that there exist constants  $A, B$  such that  $M_k \leq AM_{k-1} + B$  for all  $k \geq 1$ . We deduce that  $L_k \leq L_{k-1} + C$  for some  $C > 0$  and all  $k \geq 1$ . It follows that  $L_k \leq Ck + L_0$  and hence that

$$m(|s|^k, f) \leq L_k \leq Ck + L_0 = \frac{C}{\log |s|} \log(|s|^k) + L_0$$

for all  $k \in \mathbb{N}$ . We deduce that

$$T(r, f) = m(r, f) + N(r, f) = O(\log r)$$

for  $r = |s|^k$ ,  $k \in \mathbb{N}$ ,  $k \rightarrow \infty$ , and this implies that  $T(r, f) = O(\log r)$  as  $r \rightarrow \infty$  through any sequence of  $r$ -values. Thus  $f$  is rational.

---

*Date:* June 2, 1999.

## REFERENCES

- [1] Bergweiler, W., K. Ishizaki and N. Yanagihara, *Meromorphic solutions of some functional equations*, Method Appl. Anal., to appear

Walter Bergweiler  
Mathematisches Seminar  
Christian-Albrechts-Universität zu Kiel  
Ludewig-Meyn-Str. 4  
D-24098 Kiel  
Germany  
email: bergweiler@ math.uni-kiel.de

Katsuya Ishizaki  
Department of Mathematics  
NIPPON Institute of Technology  
4-1 Gakuendai Miyashiro  
Minamisaitama Saitama  
345 JAPAN  
email: ishi@ nit.ac.jp

Niro Yanagihara  
Department of Mathematics  
Faculty of Science  
Chiba University  
1-33 Yayoi-cho Inage  
Chiba 263  
JAPAN  
email: yanagi@ math.s.chiba-u.ac.jp