Erratum to "An example concerning iteration and factorization of rational functions" [Expo. Math. 11 (1993), 281–283]

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The paper [1] contains a crucial error – the proof of the theorem in the paper is not valid. The function R satisfying $\wp(2u) = R(\wp(u))$ is not of the form stated, but the last term in the numerator should be $32g_3z$ instead of $32g_3$. And this function can actually be factored as a composition of two rational functions of degree 2. In fact, with $P(z) = az + b + c/z$ where $a = 1/4$, $b = (e_1 + e_2)/2$ and $c = (e_1 - e_2)^2/4$ and with $Q(z) = (z - e_1)(z - e_2)/(z - e_3)$ we have $R = P \circ Q$.

But there are indeed examples of rational functions which have two factorizations into prime functions with a different number of factors. Such examples arise from elliptic functions in the cases $j = 0$ and $j = 1728$. The case $j = 0$ leads, e.g., to the example

$$z^3 \circ \frac{z^2 - 4}{z - 1} \circ \frac{z^2 + 2}{z + 1} = \frac{z(z - 8)^3}{(z + 1)^3} \circ z^3,$$

with the factors occuring being prime. From the articles of J. F. Ritt cited in [1], it seems very likely that he was aware of these examples.

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References

[1] W. Bergweiler, An example concerning iteration and factorization of rational functions, Expo. Math. 11 (1993), 281–283.